

## FURTHER TRIG EQUATION TECHNIQUES

Changing Sums to Products:

Example s:

1) Solve  $\cos 6x - \cos 2x = 0$  for  $0 \leq x \leq \pi$

$$\begin{aligned} \cos 6x - \cos 2x &= 0 \\ -2 \sin\left(\frac{6x+2x}{2}\right) \sin\left(\frac{6x-2x}{2}\right) &= 0 \\ -2 \sin 4x \sin 2x &= 0 \end{aligned} \quad \left. \vphantom{\begin{aligned} \cos 6x - \cos 2x &= 0 \\ -2 \sin\left(\frac{6x+2x}{2}\right) \sin\left(\frac{6x-2x}{2}\right) &= 0 \\ -2 \sin 4x \sin 2x &= 0 \end{aligned}} \right\} \text{ Using } \cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

So solve  $\sin 4x = 0$  and  $\sin 2x = 0$  using Graphics Calculator:

$$\sin 4x = 0: x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi. \quad \sin 2x = 0: x = 0, \frac{\pi}{2}, \pi.$$

Therefore solutions are:  $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$ .

2) Solve  $\sin 3x = \sin 2x$  for  $0^\circ \leq x \leq 360^\circ$

$$\begin{aligned} \sin 3x &= \sin 2x \\ \sin 3x - \sin 2x &= 0 \\ 2 \cos\left(\frac{3x+2x}{2}\right) \sin\left(\frac{3x-2x}{2}\right) &= 0 \\ 2 \cos(2.5x) \sin(0.5x) &= 0 \end{aligned} \quad \left. \vphantom{\begin{aligned} \sin 3x &= \sin 2x \\ \sin 3x - \sin 2x &= 0 \\ 2 \cos\left(\frac{3x+2x}{2}\right) \sin\left(\frac{3x-2x}{2}\right) &= 0 \\ 2 \cos(2.5x) \sin(0.5x) &= 0 \end{aligned}} \right\} \text{ Using } \sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

So solve  $\cos(2.5x) = 0$  and  $\sin(0.5x) = 0$  using Graphics Calculator:

$$\cos(2.5x) = 0: x = 36^\circ, 108^\circ, 180^\circ, 252^\circ, 324^\circ.$$

$$\sin(0.5x) = 0: x = 0^\circ, 360^\circ.$$

Therefore solutions are:  $0^\circ, 36^\circ, 108^\circ, 180^\circ, 252^\circ, 324^\circ, 360^\circ$ .

3) Find the general solution to  $\sin 3x + \sin 2x + \sin x = 0$ .

$$\sin 3x + \sin 2x + \sin x = 0$$

$$(\sin 3x + \sin x) + \sin 2x = 0$$

$$2 \sin\left(\frac{3x+x}{2}\right) \cos\left(\frac{3x-x}{2}\right) + \sin 2x = 0$$

$$\left. \begin{array}{l} (\sin 3x + \sin x) + \sin 2x = 0 \\ 2 \sin\left(\frac{3x+x}{2}\right) \cos\left(\frac{3x-x}{2}\right) + \sin 2x = 0 \end{array} \right\} \text{ Using } \sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$2 \sin 2x \cos x + \sin 2x = 0$$

$$\sin 2x (2 \cos x + 1) = 0$$

$$\text{So } \sin 2x = 0 \quad \text{or} \quad 2 \cos x + 1 = 0$$

$$\text{Solving } \sin 2x = 0: 2x = \sin^{-1}(0)$$

$$2x = 0 \text{ rad (this is your } \alpha \text{ value)}$$

$$2x = n\pi + (-1)^n \cdot (0)$$

$$2x = n\pi$$

$$x = \frac{n\pi}{2}$$

$$\text{Solving } 2 \cos x + 1 = 0: 2 \cos x = -1$$

$$\cos x = \frac{-1}{2}$$

$$x = \cos^{-1}\left(\frac{-1}{2}\right)$$

$$x = \frac{2\pi}{3} \text{ rad (this is your } \alpha \text{ value)}$$

$$x = 2n\pi \pm \frac{2\pi}{3}$$

$$\text{So the general solution is } x = \frac{n\pi}{2} \text{ and } x = 2n\pi \pm \frac{2\pi}{3}.$$

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